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# Markov uniqueness of degenerate elliptic operators

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**Abstract.** Let $\Omega$ be an open subset of $\mathbb{R}^d$ and $H_\Omega = -\sum_{i,j=1}^d \partial_i \, c_{ij} \, \partial_j$ be a second-order partial differential operator on $L_2(\Omega)$ with domain $C_c^\infty(\Omega)$, where the coefficients $c_{ij} \in W^{1,\infty}(\Omega)$ are real symmetric and $C = (c_{ij})$ is a strictly positive-definite matrix over $\Omega$. In particular, $H_\Omega$ is locally strongly elliptic. We analyze the submarkovian extensions of $H_\Omega$, *i.e.*, the self-adjoint extensions that generate submarkovian semigroups. Our main result states that $H_\Omega$ is Markov unique, *i.e.*, it has a unique submarkovian extension, if and only if $\mathrm{cap}_\Omega(\partial\Omega) = 0$ where $\mathrm{cap}_\Omega(\partial\Omega)$ is the capacity of the boundary of $\Omega$ measured with respect to $H_\Omega$. The second main result shows that Markov uniqueness of $H_\Omega$ is equivalent to the semigroup generated by the Friedrichs extension of $H_\Omega$ being conservative.

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